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A FAVORABLE STRATEGY FOR TWENTY-ONE*

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1. *Introduction.*—It has long been an open question as to whether those of the standard gambling games which are not repeated independent trials admit strategies favorable† to the player. There have been numerous implications[2–4] that favorable strategies do not exist. In this note, we settle the issue by showing that there is a markedly favorable mathematical‡ strategy for one of the most widely played games, twenty-one, or blackjack.

2. *Previous Work.*—Our point of departure is the work of Baldwin, Cantey, Maisel, and McDermott,[5, 6] the only serious treatment of blackjack that has been given to date. The reader will find further references and a representative set of rules in their paper. Although there are minor variations in the game, we shall adopt those rules (including insurance[6]).

3. *Method and Results.*—Our calculations are similar to those outlined in Baldwin *et al.*,[5] but there are some very important changes. First, a high-speed computer was programmed to find the player's best possible strategy and the corresponding expectation. The electronic calculator enabled us to dispense with many of the approximations that were needed by Baldwin *et al.* to reduce the calculations to desk computer size. This led to noticeable improvements in results. In particular, the player's expectation for a complete deck was found to be a startling −0.21%. (Baldwin *et al.* give −0.62%). Our second change in approach was to program the computer to do the calculations for arbitrary sets of cards. This made it possible to take into account cards that become visible during play, a feature which is essential for the determination of any winning strategy.§

A standard deck of cards has approximately 3.4×10^7 subsets which are distinguishable under the rules of blackjack. It is thus impractical to compute the optimal strategy for each of these subsets. Instead, we have studied a number of carefully preselected subsets, and from the information gained, several favorable strategies are obtained. Some of our subsets and results are given in Table 1 below.

Let Q(I) be the number of cards of value I. The special subsets in Table 1 differ from a full deck only in that the number of cards of a single value has been altered.

In actual play, these special subsets occur infrequently, and some are even impossible. Even so, they yield a profusion of winning strategies. For example, one

TABLE 1

Player's Expectation with Selected Subsets

Description of the subset	Player's expectation	Description of the subset	Player's expectation
complete deck	−.0021	Q(7) = 0	.0125
Q(1) = 0	−.0272	Q(8) = 0	.0005
Q(2) = 0	.0142	Q(9) = 0	−.0091
Q(3) = 0	.0189	Q(10) = 12	−.0215
Q(4) = 0	.0236	Q(10) = 20**	.0189
Q(5) = 0	.0329	Q(10) = 24**	.0394
Q(6) = 0	.0187		