probabilities—the relative frequency view—is particularly favored by many (perhaps most) statisticians. Is the relative frequency view of probability consistent with the one inspired by consideration of equally likely alternatives? We check the rules in Definition 3.1.

P1. First, $P(S)$ is always estimated by $N/N = 1$ so $P(S) = 1$ must be the number that the relative frequency estimates "tend to" as N increases.

P2. Since $M/N \geq 0$ always, the M/N must always "tend to" a number which is non-negative. (If, instead, the M/N got close to a negative number, then M/N would have to be negative sometimes, which is impossible.)

P3. The equation $P(E + F) = P(E) + P(F)$ is easily seen to be true when the values of $P(E), P(F)$ and $P(E + F)$ are replaced by their relative frequency estimates. Since the equation therefore holds no matter how accurately the individual terms are approximated, it must hold when the true values are inserted. (This verbal argument can be replaced by a precise mathematical proof when we later have an exact definition of "tends to.")

Thus, the probabilities assigned by the relative frequency approach agree with our earlier Definition 3.1.

The relative frequency approach gives us an experimental method for determining values for the probabilities of events. We simply repeat an experiment "enough" times so that the experimental values for the probabilities of the elementary outcomes are as close as desired to the unknown limiting or "true" values. We shall see in Chapter 3 how to determine the number of repetitions of the experiment that are required for a given accuracy.

Example 1. *Roulette*. A roulette wheel has n pockets. In Nevada, there are generally 38 pockets, numbered $1, \cdots, 36, 0, 00$. In Europe, 00 is absent and there are only 37 pockets. The pockets are intended to be of equal size and the machine is designed so that when the ball is spun, it has equal probabilities of falling into each of the n pockets. A wheel in which the n outcomes are equiprobable is **true,** as opposed to **biased.** Wheels are initially well-machined and carefully balanced to make them true. However, they sometimes become sufficiently biased so that the player who is aware of this has an advantage over the operators. Bias can be detected by recording enough trials so that the relative frequencies of the various numbers can be estimated accurately. A statistician can often detect significant bias long before it becomes apparent to the operators of the game.*

Example 2. Suppose a large horizontal disc is carefully machined and balanced, like the rotor of a roulette wheel. Given a finite probability

* An account of actual casino experiences with detecting and exploiting biased roulette wheels, and some of the mathematics involved, is given by Allan Wilson, *The Casino Gambler's Guide*, Harper and Row, 1965.