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OPTIMAL GAMBLING SYSTEMS FOR FAVORABLE GAMES[1]

by

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INTRODUCTION

In the last decade it was found that the player may have the advantage in some games of chance. We shall see that blackjack, the side bet in Nevada-style Baccarat, roulette, and the wheel of fortune all may offer the player positive expectation. The stock market has many of the features of these games of chance [5]. It offers special situations with expected returns ranging above an annual rate of 25% [23].

Once the particular theory of a game has been used to identify favorable situations, we have the problem of how best to apportion our resources. Paralleling the discoveries of favorable situations in particular games, the outlines of a general mathematical theory for exploiting these opportunities has developed [2, 3, 10, 13].

We first describe the favorable games mentioned above, those being the ones with which the author is most familiar. Then we discuss the general mathematical theory, as it has developed thus far, and its application to these games. Detailed knowledge of particular games is not needed to follow the exposition. Each discussion of a favorable game in Part I motivates a concluding probabilistic summary of that game. These summaries suffice for the discussion in Part II so that a reader who has no interest in a particular game may skip directly to the summary.

References are provided for those who wish to explore particular games in detail. For the present, a favorable game means one in which there is a strategy such that $P(\lim S_n = \infty) > 0$ where S_n is the player's capital after n trials.

PART I. FAVORABLE GAMES

1. BLACKJACK

Blackjack, or twenty-one, is a card game played throughout the world. The casinos in Nevada currently realize an annual net profit of roughly eighty million dollars from the game. Taking a price/earnings ratio of 15 as typical for present day common stocks, the Nevada blackjack operation might be compared to a $ 1.2 billion corporation.

To begin the game a dealer randomly shuffles n decks of cards and players place their bets. (The value of n does not materially affect our discussion. It generally is 1, 2, or 4, and we shall use 1 throughout.) There are a maximum and a minimum allowed bet.

The minimum insures a positive probability of eventual ruin for the player who continues to bet. The maximum protects the casino from large adverse fluctuations and in particular prevents the game from being beaten by a martingale (e.g. doubling up), especially one starting with a massive bet. In fact, without a maximum, a casino

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The paper is intended in large part to be an exposition for the general mathematical reader with some probability background, rather than for the expert.